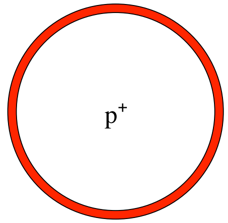
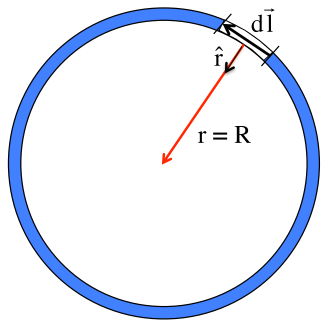
Problem 30.4

Think of the proton's motion as that of current in a ring. To determine the magnitude of the magnetic field at the center of the ring, we can use Biot Savart which is:

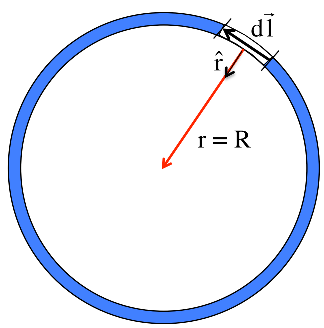


$$dB = \frac{\mu_o i}{4\pi}\frac{d\vec{l} x \hat{r}}{r^2}$$

In this relationship, $d\vec{l}$ is a differential length along the current path, $\hat{r}$ is a unit vector that defines the direction from that differential path to the point of interest, and r is that distance. In this case, those parameters are defined as shown.



Using the relationship (and noting that the sum of all the dl's around the circle is its circumference, and r is a constant quantity equal to "R"), we get:

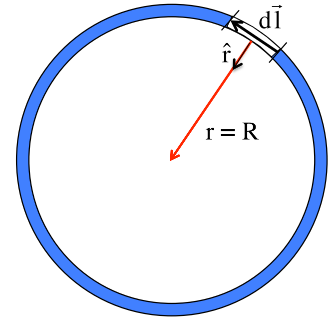


$$\begin{aligned} B &= \int dB \\ &= \frac{\mu_o i}{4\pi r^2}\int d\vec{l} x \hat{r} \\ &= \frac{\mu_o i}{4\pi R^2}\int dl \sin 90^\circ \quad (\sin 90^\circ \to 1) \\ &= \frac{\mu_o i}{4\pi R^2}(2\pi R) \\ &= \frac{\mu_o i}{2R} \end{aligned}$$

All we need is the current expression.

The current is:

$$i = \frac{\Delta q}{\Delta t}$$



If Δt is the time it takes the electron to make one trip around the ring, we can write the velocity as:

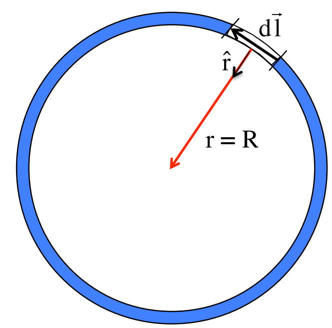
$$v = \frac{2\pi R}{\Delta t}$$

$$\Rightarrow \quad \Delta t = 2\pi R / v$$

Coupling this with the expression for current and we get:

$$\begin{aligned} i &= \frac{\Delta q}{\Delta t} \\ &= \frac{\Delta q}{\left(2\pi R / v\right)} \\ &= \frac{ev}{2\pi R} \end{aligned}$$

Coupling this with our Biot Savart derived expression and we can write:



$$\begin{aligned} B &= \frac{\mu_o i}{2R} \\ &= \frac{\mu_o\left(\frac{ev}{2\pi R}\right)}{2R} \\ &= \frac{\mu_o ev}{4\pi R^2} \\ &= \frac{(4\pi x10^{-7}\ T\bullet m/A)(1.6x10^{-19}\ C)(2.19x10^{6}\ m/s)}{4\pi(5.29x10^{-11}\ m)^2} \\ &= 12.5\ T \end{aligned}$$